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# Generative Structural Design for Embodied Carbon Estimation

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## Abstract

This research proposes a new strategy for estimating the embodied carbon in existing residential and commercial buildings. Most critical in this estimation is the structural system, up to two thirds of the whole embodied carbon of a building. Existing benchmarks rely on area normalized carbon content values that do not take the scaling effects of spatial structures – both horizontally and vertically – into account. The embodied carbon of a structure can be greatly over- or underestimated based on its effective size, when assessed with square meter averages or typological approaches from existing databases. We introduce an automated workflow that leverages generative structural design for the creation of a structurally-sound building model. Specifically, our method is applicable for large scale steel framing systems that are widely used for commercial developments. With this generative approach, we can accurately and instantly calculate the structural quantities and embodied carbon of a structural framing system without the need for manual input. The method is scalable to urban-level assessment. We show how calibrated simulation results achieve a sub 17% error margin when compared with real-world buildings.

**Keywords**: Embodied Carbon, Analysis, Generative Modelling, Housing Stock, Steel Structures, Steel Framing, Carbon, Hangai Prize

## 1. Introduction

In the coming three decades, over 226 billion square meters of buildings are projected to be built worldwide – a doubling of the global building stock [1]. With construction and energy use of buildings already accounting for almost 40% of current carbon emissions [2], there is an immediate need for new strategies that combine net zero energy building operation with net zero carbon construction practices. Combined, these two approaches have the potential to save over 150 GT$CO_2$ emissions over the coming 30 years (Figure 1) – up to a third of the current carbon budget [3]. While the bulk of previous efforts focused on reducing operational energy use, the figure underlines that we must start at decarbonizing the very foundations of buildings before they are built.

In places such as Europe and the United States – where over two thirds of the anticipated building stock by 2050 is already built – the fight against emissions in the built environment will largely be focused on renovation of existing buildings and cities [1]. Detailed case studies for the redevelopment of a landmark building have revealed the enormous $CO_2$ savings that can be achieved by renovation instead of building from scratch [4]. In certain cases, a retrofit can save more carbon than a newbuild in its entire lifetime. This emphasizes the importance of assessing embodied carbon in the existing building stock to better inform development decisions on both the building and urban scales, in addition to estimating carbon impacts of early-stage design decisions for new construction.

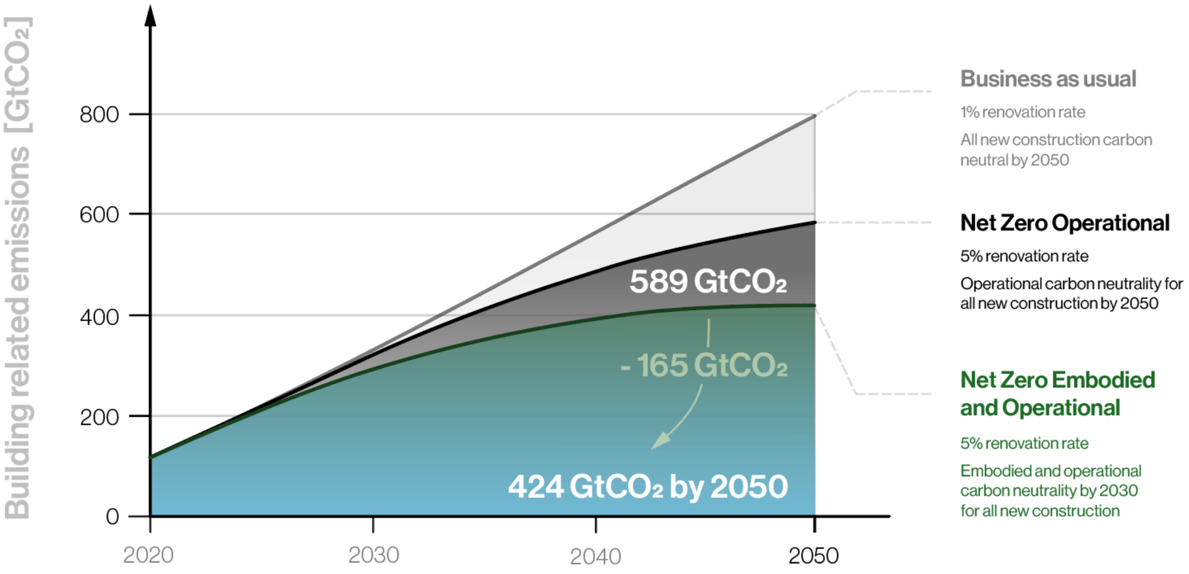


Figure 1: Excel model of building related emissions from buildings in the years 2020-2050. Embodied carbon emissions of 410 $kgCO_2/m^2$ for existing and new built and 100 $kgCO_2/m^2$ for retrofits are assumed. The *Net Zero Operational* assumes a fully decarbonized grid by 2050 and *Net Zero Embodied and Operational* assumes an additional linear decrease of embodied carbon from 2030 to 2040 to 0. Global floor areas and operational emissions are based on projections by the International Energy Agency (IEA) [1].

Better strategies are needed for both estimation and reduction of embodied carbon in current and future buildings. Existing benchmarks are inconclusive, using embodied carbon values with a wide range of carbon content, ranging from 300-1650 $kg_{CO2e}/m^2$, depending on the source [5]. This significant uncertainty and variance originates from highly diverse databases and statistical averages of general housing stock or varying building archetypes [6]. More accurate surveys can inform new strategies for minimizing embodied carbon in early design stages and can inform decision making on building retrofits and material choices.

A net zero carbon production of building materials comes with significant technical challenges in availability and scalability of sustainable materials (such as timber) and production methods, that in many cases are not yet economically feasible, such as renewable steel or concrete production. This increases the importance of design strategies that can have a massive impact on embodied carbon: increased structural efficiency, optimization strategies for utilization of less material, reusability of materials [7], more economic and adaptive usage of space and longer lifespans of structures – building more with less [8].

Different computational strategies have been proposed to calculate and best estimate the embodied carbon impact of buildings. They have largely focused on surveys of recently constructed buildings, where building specifications and material quantities are already known. Where available, building information models (BIM) can be combined with a suite of specialist computational tools and material databases to assess their embodied carbon. Such an analysis can inform different material choices and design decisions in the later planning stages to reduce the embodied carbon [9]. When deployed on a larger scale, for city redevelopment or masterplans, accurate modelling of existing buildings is often not feasible, as it would require the manual creation of 3D BIM models and on-site surveys by specialists.

To estimate the embodied carbon of un-built or un-surveyed buildings, area calculations from massing models can be multiplied with area normalized benchmarks. More detailed data on building elements such as façades can be included when specified in a building's massing. However, area normalized estimation of a building's structure can be highly problematic. An analysis of multi-story concrete residential buildings in India showed more than 60% of the embodied carbon to be from the structure [10]. In the case of steel framed buildings the embodied carbon database EC3 reveals for their "Commercial Core & Shell - Steel Example" building, that 49% of total embodied carbon emissions are from the steel structure alone and over 66% when including structural concrete elements [11]. These structural material quantities do not scale linearly with size and are dependent on myriad factors including construction method, loads and building size.

We identify a gap between fuzzy benchmark numbers and high-resolution BIM models for embodied carbon estimation and thus introduce in this paper a hybrid approach to measure embodied carbon, specifically of steel-framed buildings, using generative structural design and sizing optimization. As outlined in Figure 2, our algorithm takes building massing and automatically dimensioned structural elements for the embodied carbon calculation, while relying on proven methods for the building envelope, creating a proxy parametric building model that serves as a simplified BIM model. A comparative study with real-world building data shows how our workflow can estimate structural material quantities comparable to real-world building data. The fully automated nature of the model further allows for its implementation with existing carbon estimation tools in urban modelling software and its use with surrogate modelling and machine learning algorithms in the future.

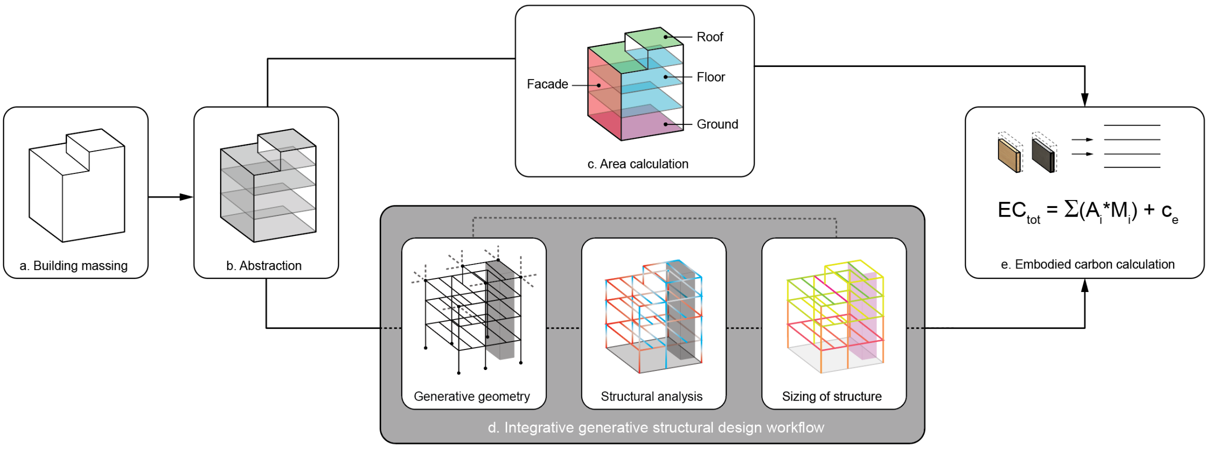


Figure 2: Proposed workflow for embodied carbon calculation: Building massing (a.), geometric abstraction (b.), area calculation of linearly scalable building elements (c.), integrative generative design geometry generation analysis and sizing of structural members (d.) and final embodied carbon calculation (e.).

## 2. Background

While methods for estimation of operational energy on both building and urban scales are relatively well established [8], there can be a high degree of uncertainty in prediction of the embodied carbon of a building. We can differentiate between benchmark datasets and material quantification methods.

Material quantification methods rely on an accurate tally of building materials used in a building that is then then multiplied by their specific carbon content. This is typically done via spread sheet-based tools, which are standalone and generic databases that serve as lookup tables for embodied carbon. As public or privately maintained databases, they include building materials, sometimes with associated suppliers and reference projects that encourage savings [11]–[14]. A database such as EC3 [11] shows that it is not only crucial to choose material with inherently low carbon emissions, but also a manufacturer itself with a low carbon supply chain. The more thoughtful choice of manufacturer for building materials can cut emissions significantly.

A number of CAD-integrated tools plug directly into architectural design workflows and connect architectural 3D modelling software to spread-sheet based databases [15]–[18]. This allows for an automated tally and highly accurate estimation of the embodied carbon. However, since exact material quantities must be known, such estimates are only possible at end of the design stage for a building when fundamental changes in structure and global design are hard and costly.

For estimation of embodied carbon of buildings where exact material specifications are unknown, we must rely on benchmark datasets. The estimated benchmark value is a normalized area value of $kgCO_2$/m2 as a best guess. Additional information on location, building type, size and height, and structural system material increases the accuracy of the prediction [19]. However, buildings typologies and use type alone do not give an accurate estimate for the embodied carbon per square meter and come with significant uncertainty [20]. Datasets with small number of projects cannot take local construction methods and material supply into account and are not yet available and applicable on a global scale. The high ranges and uncertainties stress the importance of more detailed information on a building's construction material or a full BIM model of a building.

## 3. Methods

Our proposed physics-based estimation of embodied carbon creates a generative model of a building's structure for material quantification – a quasi-BIM model of a building's structural elements. In steel framed buildings optimal relationships between structural primary girders and secondary beams have been widely explored [21], resulting in rules of thumbs that have commonly used today. To further investigate the inherent relationship between primary and secondary structural members we analytically compute the relationship between spans and structural material quantity converted to embodied carbon in Figure 3. Derived from an analytical equation a simple beam model with prismatic members of rectangular (concrete, timber) or I-shaped (steel) cross sections optimally sized for typical loading shows the inherent relationship between geometric subdivision and spacing of members.

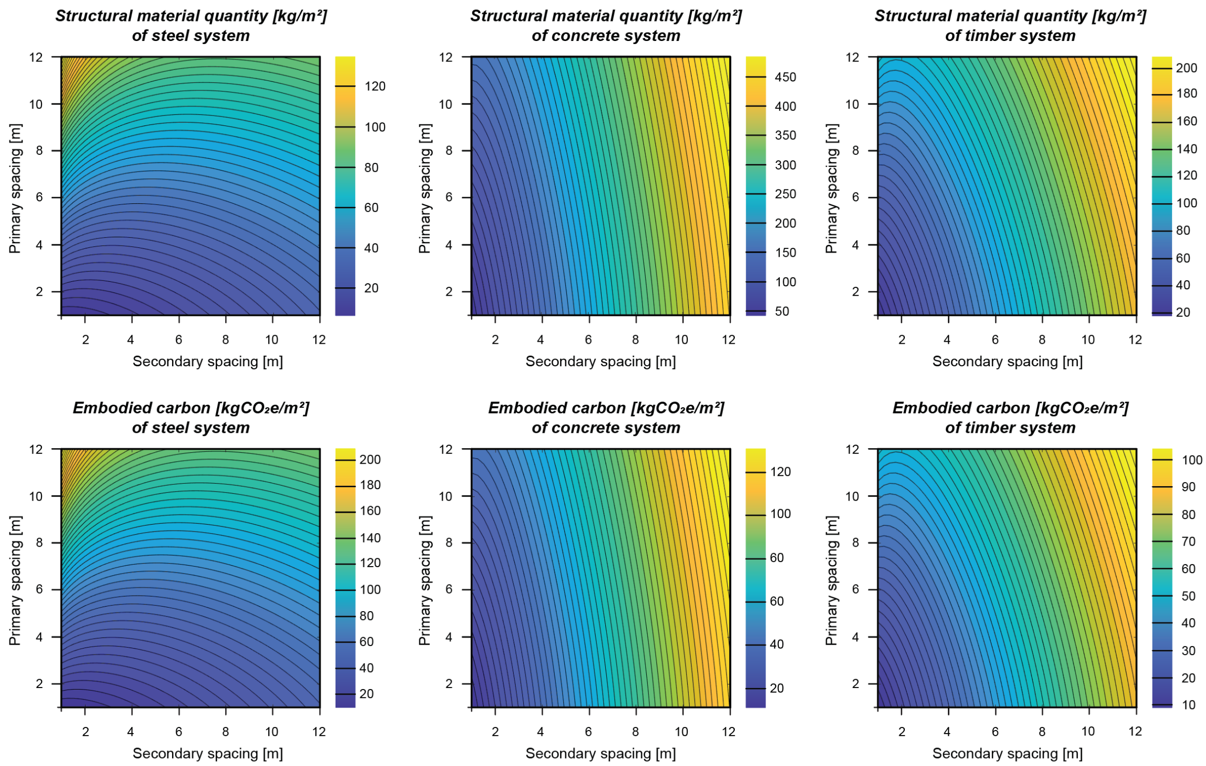


Figure 3: Structural material quantity and embodied carbon of steel, concrete and timber system with differentiated primary and secondary member spacing. Embodied carbon values are computed by multiplying the structural material quantities by embodied carbon coefficients: 0.50 for timber (glulam beams and CLT panels), 1.55 for steel (I-shaped structural sections), and 2.00 for concrete (beams with one-way flat slabs, 2% reinforcement ratio), selected from the 2019 ICE Database v3.0 [22].

The relationship across steel, concrete and timber systems shows the different behaviour of the materials and the cost of primary span, most significantly affecting steel structures.

In our more detailed analysis for real world buildings, we chose to focus on buildings constructed with steel framing, a construction system that is widely used for standard commercial developments of large-scale office and residential buildings. In this research we specifically investigate estimating structural material quantities of the main structural floor framing of such buildings, which is a major component of total structural material and varies widely based on geometry and material decisions. We create a geometric layout of the steel framing system, which we dimension using an assumed load derived from building codes. Without knowing the actual geometry of a building's structure, through optimizing the generative geometry model towards low weight while incorporating constraints of clear span and loads, we create a fully dimensioned structural system for any building massing.

To test and calibrate the generative models, we relied on a data set of four steel frame buildings, as adapted from Tan [23]. A single floor of the steel framing plans with dimensioned cross sections and loading serve as the benchmark for our simulation. As our testing framework, we used the 3D modelling package Rhinoceros 3D with its parametric node based visual scripting environment Grasshopper [24]. Custom scripts were combined with optimization framework DSE [25] and the structural solver Karamba3D [26].

The four buildings feature different architectural typologies as shown in Figure 3; Building #1 is an office tower, building #2 a school, building #3 a warehouse, and building #4 a university. Through this typological diversity, the clear spans range from 2 to 16.5 meters. Cores and walls were abstracted as supports while steel columns were included in the model.

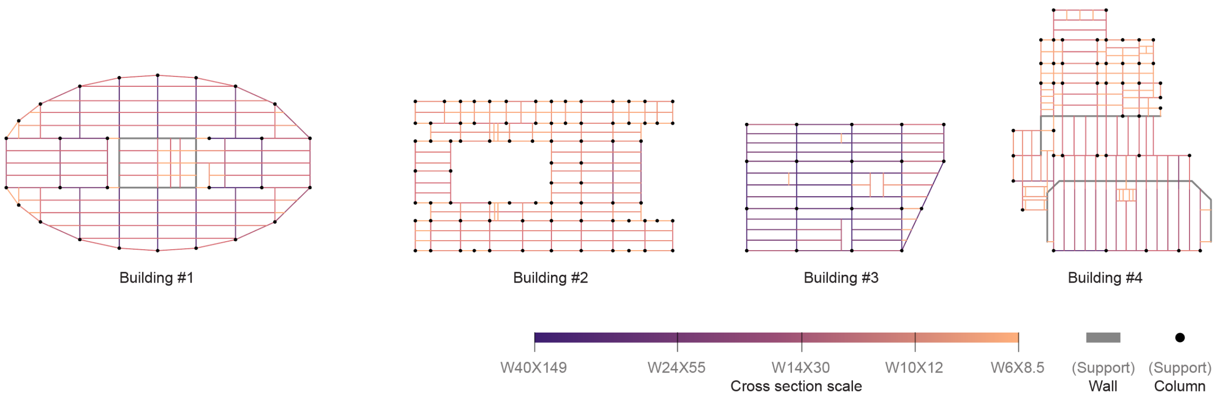


Figure 4: Steel framing plans of initial case study dataset Building# 1-4

Based on our benchmark data sets, a façade load of 500 plf (0.68 kN/m) was applied to the perimeter beams as well as a dead load for the concrete deck of 45psf (2.2 kN/$m^2$) and an additional superimposed dead load for finishes and equipment of 20 psf (1kN/$m^2$). Live loads vary based on programmatic requirements referenced from ASCE 7-10 Table 4-1 [27] and sum up to to a total of 125 psf (6 kN/$m^2$) for building #1, 115 psf (5.5 kN/$m^2$) for building #2, 315 psf or (15.1 kN/$m^2$) for building #3 and 145 psf or (6.9 kN/$m^2$) for building #4.

Our reference steel framing data set relies on a series of detailed structural design assumptions, such as cambering of the steel beams and concrete slab on steel deck that works compositely with the beams; both of these inclusions increase the steel beams' structural capacity. We simplified these features, as they are not implemented in the structural solver. We therefore adjusted our benchmark model to differ slightly from the real-world dataset by using the real-world geometry with optimized cross sections as the comparison benchmark for the generative system. A maximum displacement of maximum beam length divided by 120 (instead of 240) was chosen to account for these differences. Resulting in a maximum displacement of 6.2 cm (#1 with 14.7m max. span), 5.4 cm (#3 with 12.8m max. span), 6.9 cm (#4 with 16.5m max span) and 3.2 cm (#2 with 7.6m max. span).

For the geometric generation of a beam layout, we propose two generative methods; the *Voronoi* method and the *Cut Out* method. As visualized in Figure 5 both methods produce quasi optimal solution averages that predict the weight of the structure. The *Voronoi method* subdivides the floor into equal areas, and the number of divisions, which defines the bay size, and the beam spacing can be input parametrically. The *Cut out* method creates a rectangular grid with variable bay sizes in *x* and *y* direction and adjustable column cadence that is cut out from the boundary curve of the original floor plate. After generating the geometry, the structural members are split up into a hierarchy of girders and beams which is reflected in their structural simulation. After generating the geometry, we apply the loading of the original building, including area and façade load, to the structure and automatically size cross sections based on Eurocode. The final volume of the beam geometry is measured and returned as total mass per m$^2$. Each of these methods can be optimized or sampled over the input parameters, which generate a range of results.

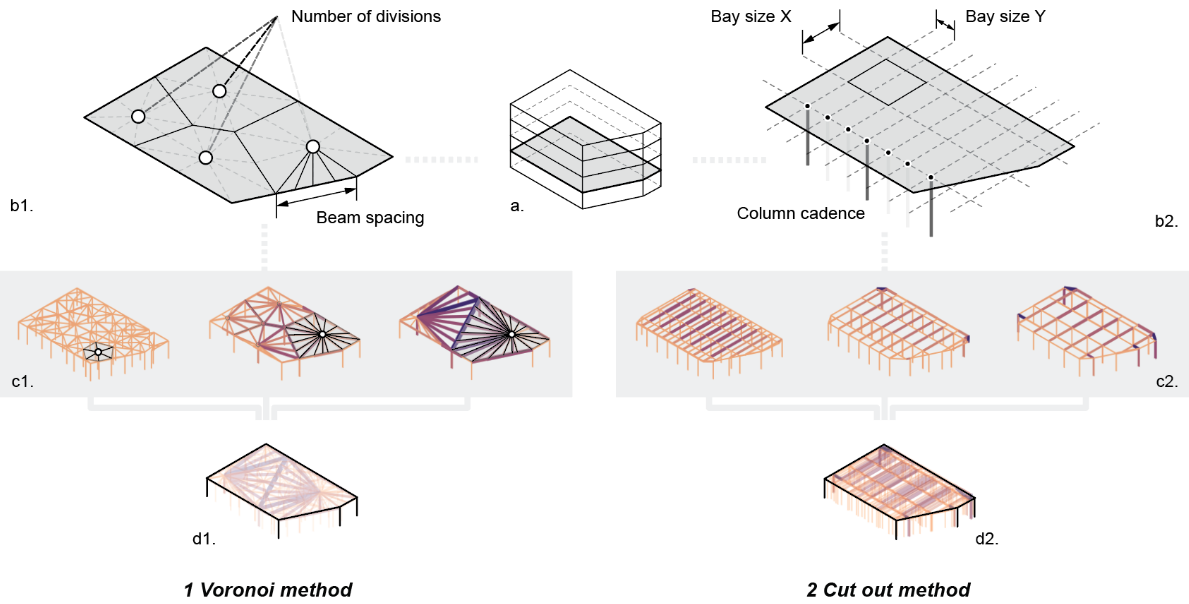


Figure 5: *Voronoi* (1) and *Cut Out* (2) method take a floor slab of an existing massing as an input (a.). Both methods adaptively subdivide the floor area (b1., b2.) to create a variety of layouts with differentiated bay sizes and beam spacings (c1., c2.). An average of quasi optimal solutions leads to the predicted weight and sizing of the structure (d1., d2.)

## 4. Results

The following section describes the results of the Voronoi and Cut out method applied to our building dataset, as well as sampling larger design spaces with parametric variation of the input variables.

When applied to our reference buildings (#1-4) the results of both the *Voronoi* and the *Cut out* method are described in Figure 6. To study the structures more consistently, the former US cross sections were adapted their closest fitting European counterpart and converted to HEA/HEB/HEM /HEAA as our structural solver works with European sizing code. This differs from the original weights due to differences in sizing from US to Eurocode. The conversion is described as observed building (a.) with total mass ranging from 40 to 76 kg/m$^2$. To benchmark and compare the geometric creation methods and the sizing algorithm we additionally calculate the optimal sizing based on the original geometry (b.).

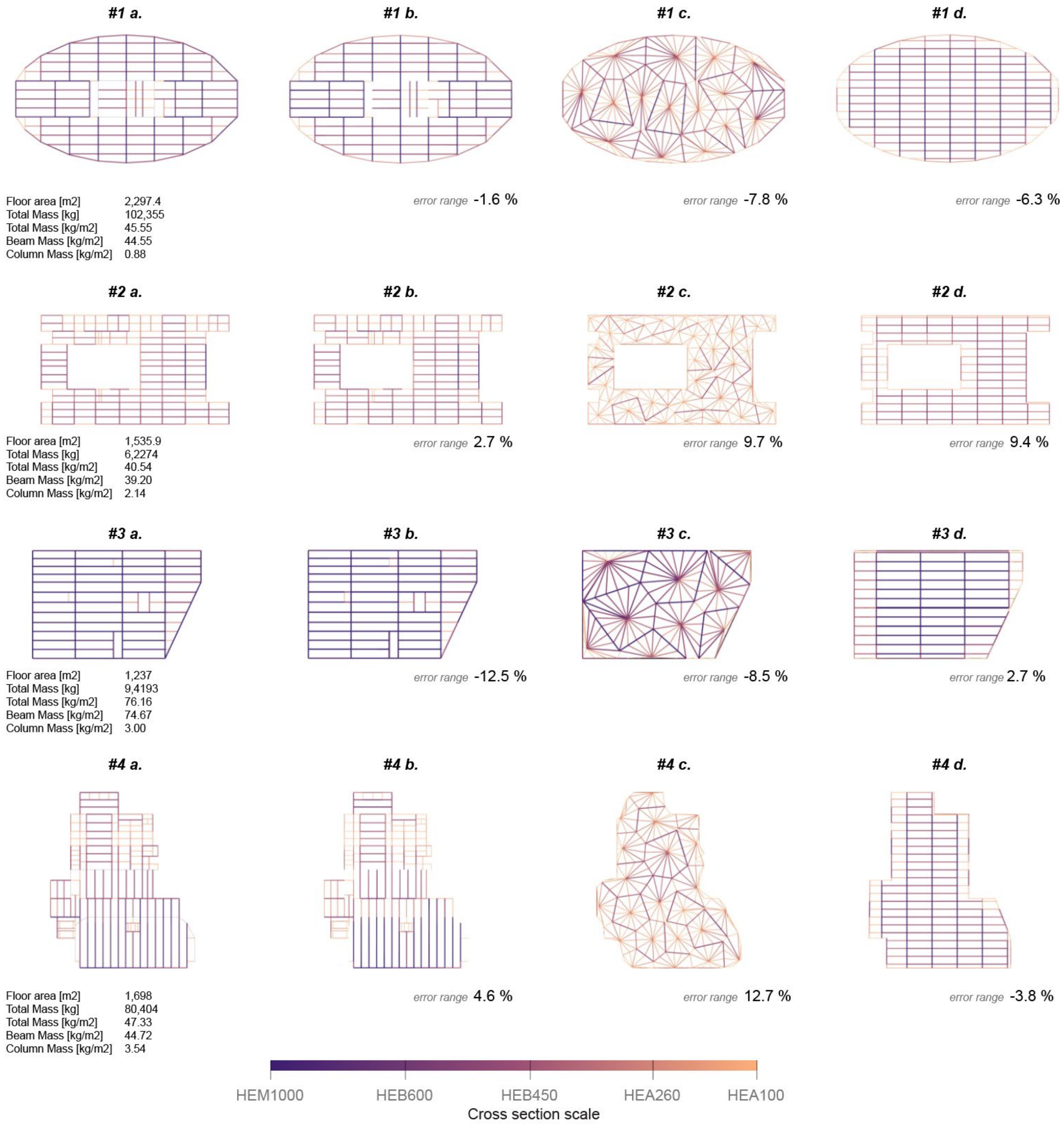


Figure 6: Buildings #1-4 with the observed building (a.), an optimal sizing of members based on real geometry (b.), optimal sizing of members based on *Vorono*i method (c.) and *Cut out* method (d.).

The optimal sizing should return results as close to the original as possible to best reflect the built structure, resulting in error ranges from 1.6% to 12.7%, reflecting the simplification of cambering and composite action from the original dimensions.

For the Voronoi method (c.) the average girder span of the original building and a bay size of 3m was chosen as input parameters. This resulted in bay sizes of 10.68m (#1), 5.7m (#2), 10.16m (#3) and 8.2m (#4). Using the girder average length as the input variable proved to be appropriate for the more regular structures #1-3 while the large variety of spans of the lecture hall of the university building #4 with 16.5m building #4 caused a slight distortion. Error ranges fall under 10% in buildings #1-3 and 12.7% in building #4.

The *Cut out* method (d.) was used with input parameters most closely representing the original building. For buildings #1,2 and #4 the grid size was set at 8x2.5m with columns every corner in X direction and every 5$^{th}$ beam in Y direction. For #3 longer spans of 12x2.5m and columns, every corner in X and every 2$^{nd}$ in Y direction were chosen. The warehouse building, with it is high load, stressed the maximum standardized cross section and artificially larger cross sections had to be provided for the sizing algorithm to find a solution.

To further study the parameter space of the two methods, a series of 1000 samples was calculated with random input samples, applying both *Voronoi* and *Cut out* method to each building geometry, as shown in Figure 7. The models were calculated with varying seed values for subdivision point placement, bay sizing (3-15m), beam spacing (3m) in the *Voronoi* method and varying *x* and *y* grid length (2-20m) and fixed column cadence (2) in the *Cut out* method. The scatter plots display the Pareto front and the median axis of the real buildings and the original structural quantity in kg per m$^2$ of the reference structure.

The structural material quantities fall in a 10% range of generated values from our real building values when the bay size of the original building is known. Averaged over +- 0.5m bay size, the results stay within a 25% margin of the average samples generated with the Voronoi method (#1, 1.1%, #2 17%, #3 3.6%, #4 24.5%) and within a 17% margin with the *Cut out* method (#1 5.8%, #2 15.6%, #3 16.5%, #4 9.2%).

## 4. Discussion and conclusions

Structural systems are one of the most carbon intensive parts of a building and therefore a crucial component in assessing a building's embodied carbon. Given the difficulties posed by the reduction of carbon emissions to the building industry's efforts to combat climate change, better modes of analysis and prediction can help us to gain a better understanding of existing housing stock. Furthermore, the relationships between broad structural material quantity loading, architectural programs and geometry can inform future construction systems and design decisions. Our method presents a first step towards analysing a housing stock based on external geometry and can take advantage of large geometric GIS datasets with building massings available for cities and buildings worldwide.

Currently, a building's embodied carbon can be assessed either via a benchmark database or a full material survey. Current databases are sparse and limited in their architectural program, location, typology, and construction system and therefore come with significant uncertainty. There are no big public repositories, and the existing databases are small. As De Wolf writes; "Industry lacks the appropriate benchmarks to know how much materials are needed for various structures." [19]. Proprietary and with a wide range of low carbon to commercial buildings, making them difficult for benchmarking. A full life cycle analysis (LCA) is always done after the fact; a full material survey requires a full 3D BIM model that is only available at the end of the design process, after the most important design decisions are difficult to adjust or require a laborious accounting of an existing structure by specialists.

One of the main sources of uncertainty is a building's structure. Compared to material quantities of building envelopes, it is hard to estimate based on a building massing. We propose a physics-based method for estimating embodied structural material quantities of steel frame structures. A generative geometry workflow creates a mock-up structure from a given building outline that is fully dimensioned using realistic loading conditions and cross sections. This creates a simplified structural model of a steel framing system we can further use to analyse a building's embodied carbon.

Generative design algorithms that are typically used for the creation of novel buildings are proposed to reverse engineer structural components of existing structures. Our unique approach offers two methods for generating geometry and structural material quantities. Due to the limited dataset, it is unclear if one method can perform significantly better. Our results show that all structures are highly sensitive and rely on a correct input of loading and span.

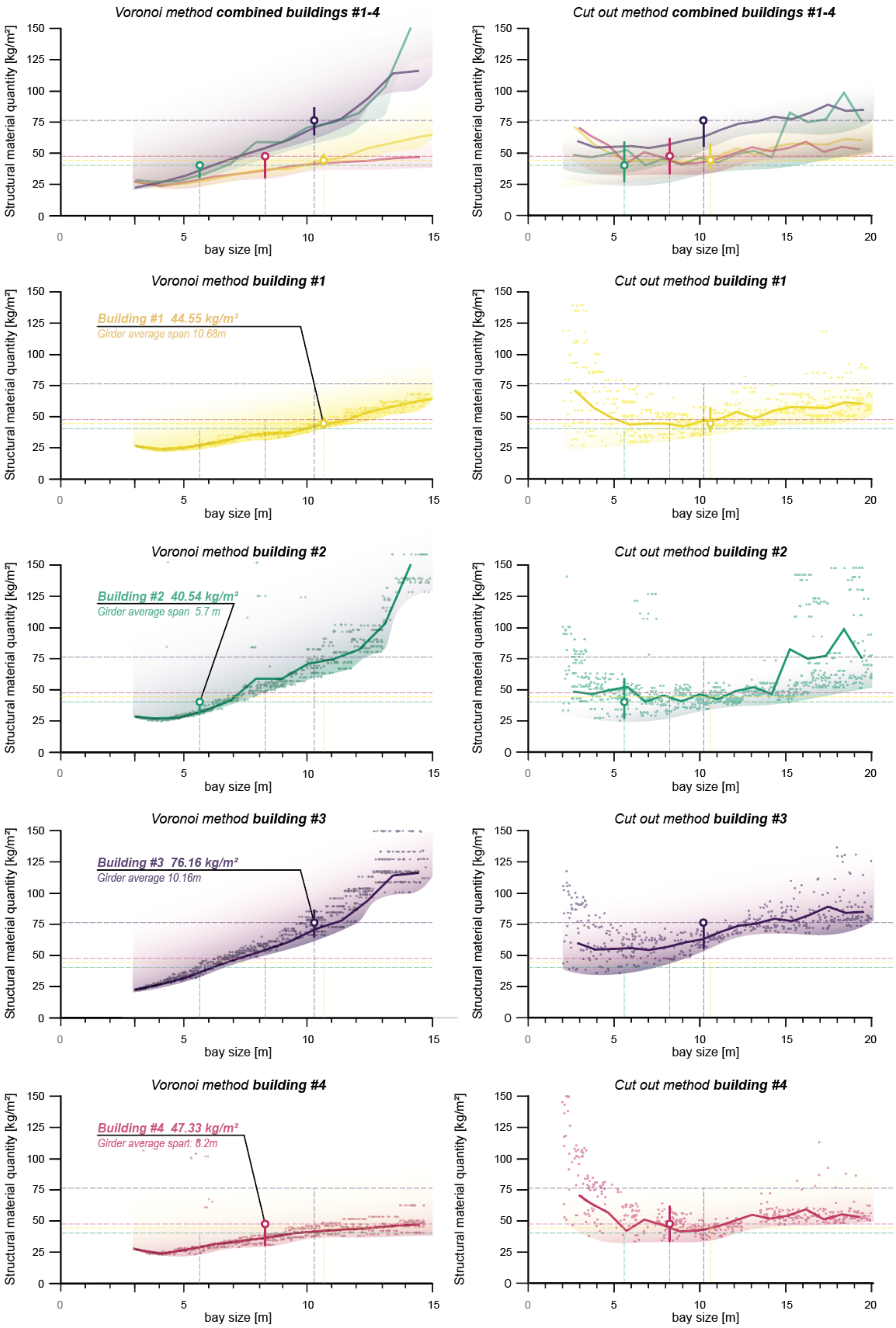


Figure 7: Comparison of 1000 samples of Voronoi and Cut out method on the #1-4 Building Dataset.

The regular office building (#1) and school building (#2) with short spans show prediction errors of under 10%. Special cases can pose a challenge for the algorithmic prediction, such as the warehouse building (#3) with large spans that go to loading limits for conventional steel cross sections. Furthermore, the university building (#4) with a large lecture hall shows how long spans can have a high impact and distort averages.

The method is currently limited by its inputs – bay size, materiality, and loads – that needs to be assessed beforehand. GIS information such as zoning, the year the structure was built, and requirements of loading per building code can help define these inputs on a broader scale.

The results of both our cross-material analysis and the building dataset show how we do not need precise geometry for minimal embodied carbon, as scale and spans are the decisive factors. While a number of geometric configurations are not efficient, there is still a lot of architectural freedom for the design of low-embodied carbon structures. The flat design space suggests that various complex load paths can create an efficient structure. Our findings are supported by statistical analysis of the deQo Database that suggested a correlation between span and embodied carbon is crucial in determining a building's embodied carbon and is far more important than floor area or building exterior massing [19]. This trade-off between large spans and structural material quantities is clearly visible and suggests that "open floor plans" with inherently more material should be carefully evaluated for their architectural trade-offs. Competing motivations of future spatial flexibility and low embodied carbon must be further assessed and studied more in depth.

The results and analysis of the structural framing plans show a large variety of structural material quantity over the four sample buildings, almost doubling the embodied structural material quantities, and thus embodied carbon, based on different spans and loads. The analysis of our parameter space shows that a correct estimation of the bay size of the real structure allows for an estimation of the buildings structural material quantity using our two methods.

In this paper we present a first proof of concept for a novel generative design workflow for embodied carbon analysis. Future work utilizing larger calibrated building datasets will be required to make embodied carbon predictions with high accuracy. Further assessing the performance and refining our geometric methods. An integration with secondary geometric details for building envelopes floors and cores will allow for a comprehensive study of embodied carbon. Additional material systems widely used in construction such as concrete, brick and timber systems would have to be investigated to make predictions about larger urban building stock. For example, the inclusion of lateral systems in the simulation would enable predictions of tall buildings. As scaling effects have a great effect on skyscrapers exposed to wind loads, which is reflected in non-linear increase of their structural mass with greater height [28].

A more precise estimation of a building's internal geometry could further enable more comprehensive operational energy simulations, incorporating previously unknown variables such as thermal mass or internal layout. Engineers and architects make key LCA design decisions during schematic design development. The implementation of fast analytical and predictive tools in design environments could enable more informed early-stage design decisions. Accurate prediction of embodied carbon is crucial for decisions regarding existing housing stock. From the small scale – informing stakeholders towards retrofit decisions and better estimate impact of new real estate developments – and on a larger scale – guiding city scale building code and policy implementations for embodied carbon building standards and large-scale masterplans.

## Acknowledgements

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